FORECASTING FUNDAMENTALS

Forecast: A prediction, projection, or estimate of some future activity, event, or occurrence.

Types of Forecasts

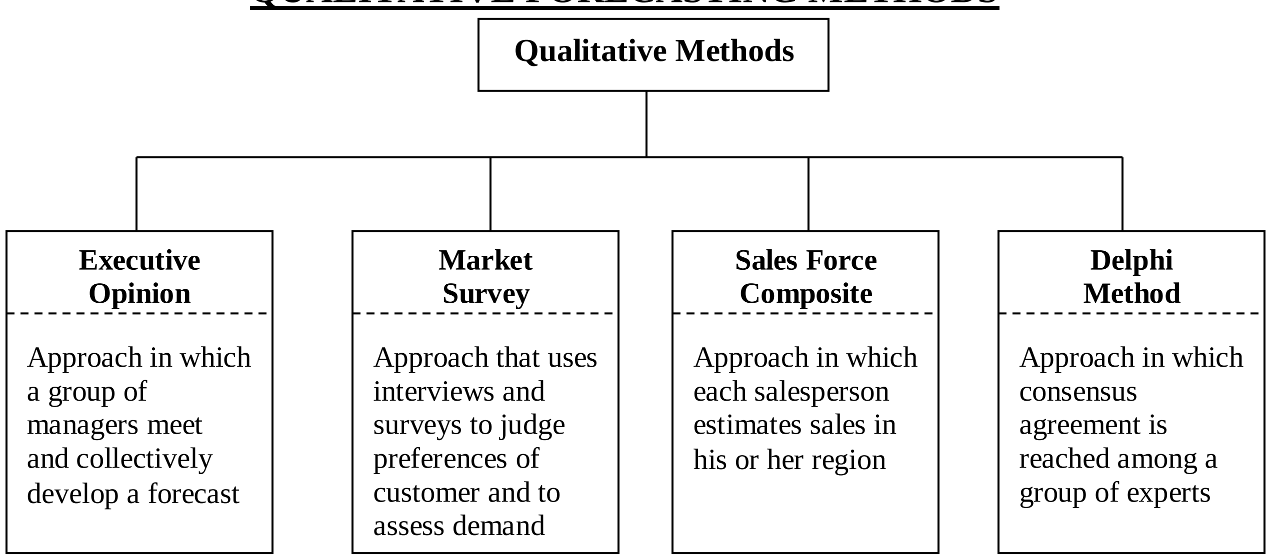
- Economic forecasts
  - Predict a variety of economic indicators, like money supply, inflation rates, interest rates, etc.
- Technological forecasts
  - Predict rates of technological progress and innovation.
- Demand forecasts
  - Predict the future demand for a company's products or services.

Since virtually all the operations management decisions (in both the strategic category and the tactical category) require as input a good estimate of future demand, this is the type of forecasting that is emphasized in our textbook and in this course.**TYPES OF FORECASTING METHODS**

Qualitative methods: These types of forecasting methods are based on judgments, opinions, intuition, emotions, or personal experiences and are subjective in nature. They do not rely on any rigorous mathematical computations.

Quantitative methods: These types of forecasting methods are based on mathematical (quantitative) models, and are objective in nature. They rely heavily on mathematical computations.

QUALITATIVE FORECASTING METHODS



QUANTITATIVE FORECASTING METHODS

Quantitative Methods

Time-Series Models

Time series models look at past patterns of data and attempt to predict the future based upon the underlying patterns contained within those data.

Associative Models

Associative models (often called causal models) assume that the variable being forecasted is related to other variables in the environment. They try to project based upon those associations.

TIME SERIES MODELS

Model	Description
Naïve	Uses last period's actual value as a forecast
Simple Mean (Average)	Uses an average of all past data as a forecast
Simple Moving Average	Uses an average of a specified number of the most recent observations, with each observation receiving the same emphasis (weight)
Weighted Moving Average	Uses an average of a specified number of the most recent observations, with each observation receiving a different emphasis (weight)
Exponential Smoothing	A weighted average procedure with weights declining exponentially as data become older
Trend Projection	Technique that uses the least squares method to fit a straight line to the data
Seasonal Indexes	A mechanism for adjusting the forecast to accommodate any seasonal patterns inherent in the data

DECOMPOSITION OF A TIME SERIES

Patterns that may be present in a time series

Trend: Data exhibit a steady growth or decline over time.

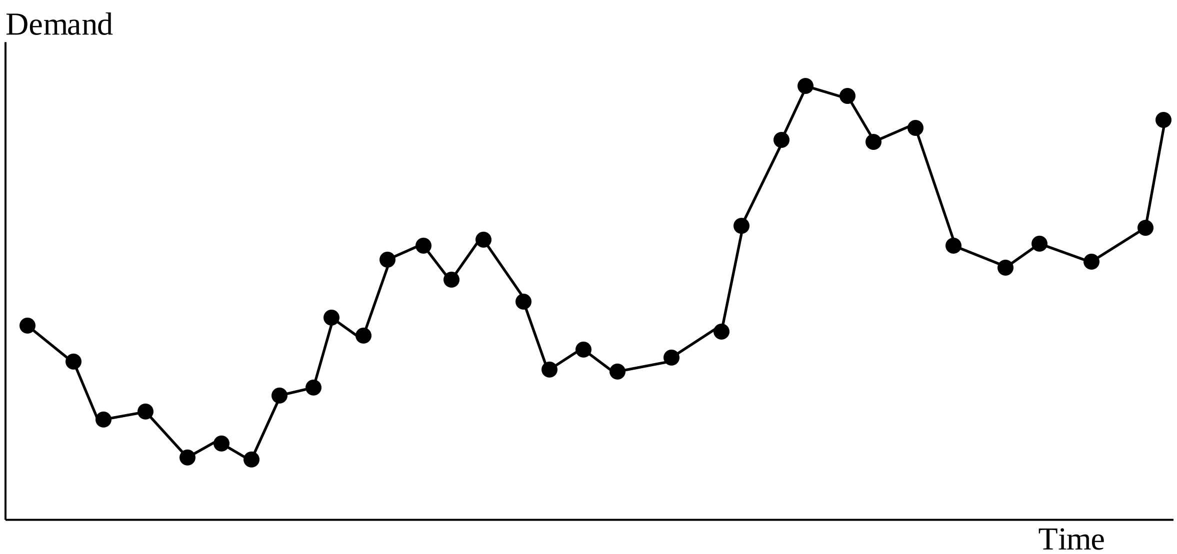
Seasonality: Data exhibit upward and downward swings in a short to intermediate time frame (most notably during a year).

Cycles: Data exhibit upward and downward swings in over a very long time frame.

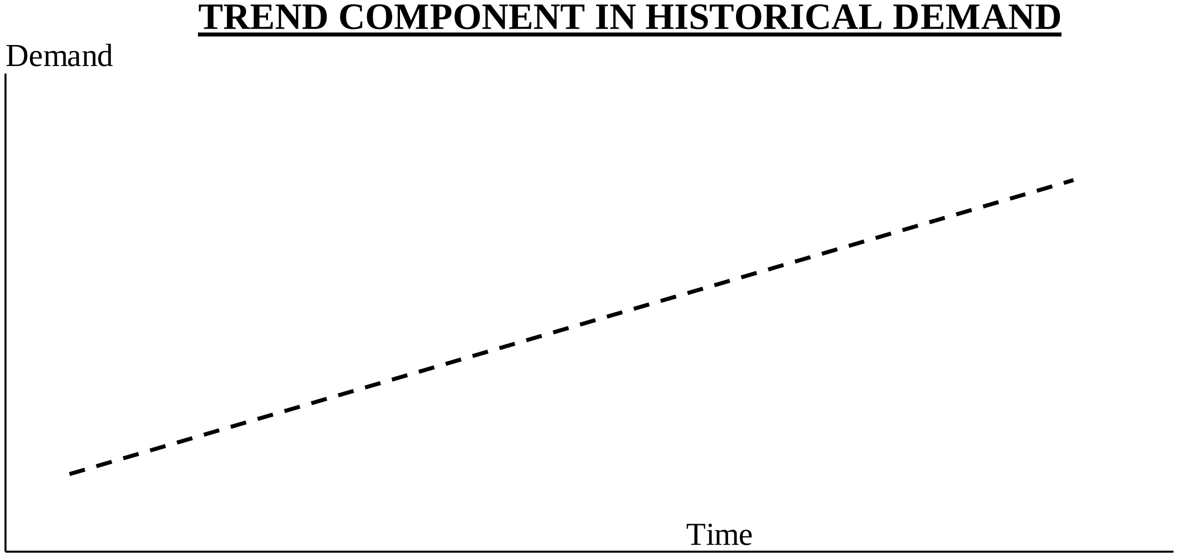
Random variations: Erratic and unpredictable variation in the data over time with no discernable pattern.

ILLUSTRATION OF TIME SERIES DECOMPOSITION

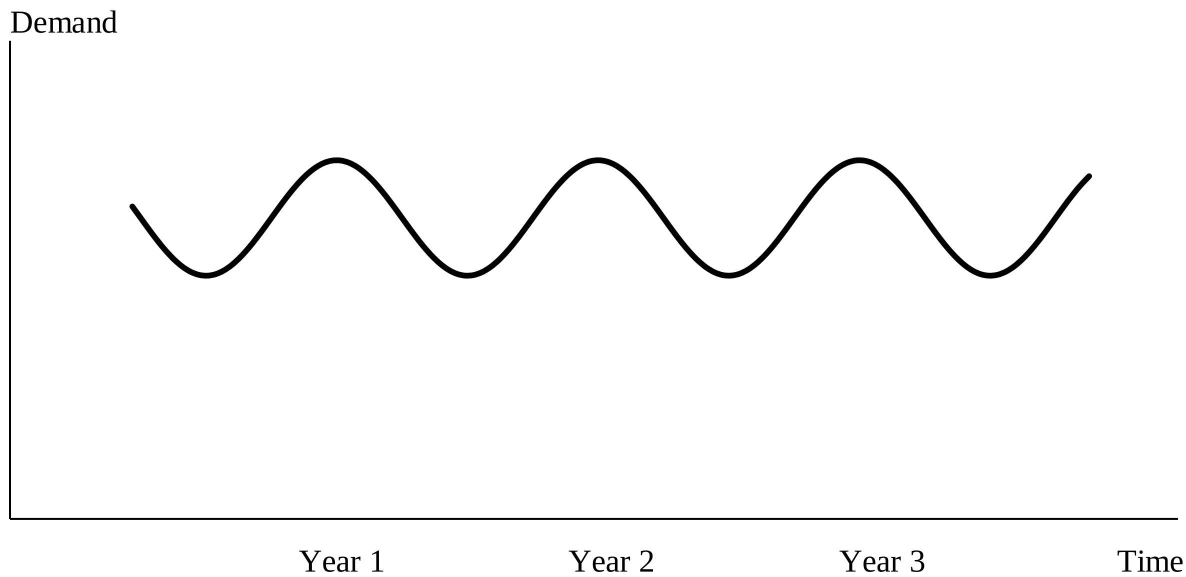
Hypothetical Pattern of Historical Demand



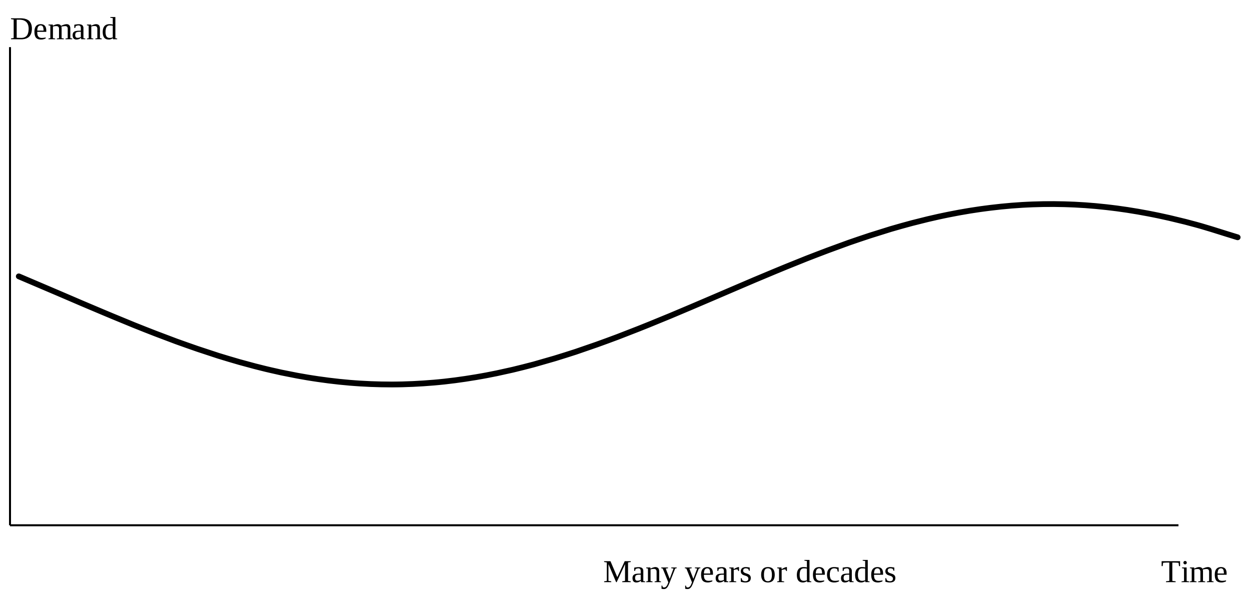
TREND COMPONENT IN HISTORICAL DEMAND



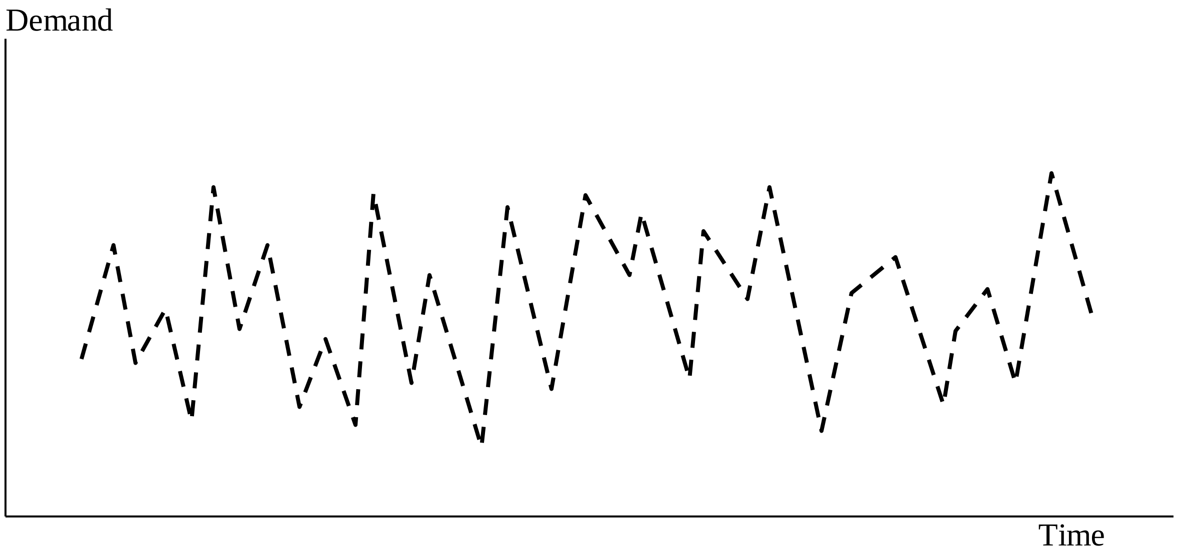
SEASONAL COMPONENT IN HISTORICAL DEMAND



CYCLE COMPONENT IN HISTORICAL DEMAND



RANDOM COMPONENT IN HISTORICAL DEMAND



DATA SET TO DEMONSTRATE FORECASTING METHODS

The following data set represents a set of hypothetical demands that have occurred over several consecutive years. The data have been collected on a quarterly basis, and these quarterly values have been amalgamated into yearly totals.

For various illustrations that follow, we may make slightly different assumptions about starting points to get the process started for different models. In most cases we will assume that each year a forecast has been made for the subsequent year. Then, after a year has transpired we will have observed what the actual demand turned out to be (and we will surely see differences between what we had forecasted and what actually occurred, for, after all, the forecasts are merely educated guesses).

Finally, to keep the numbers at a manageable size, several zeros have been dropped off the numbers (i.e., these numbers represent demands in thousands of units).

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Total Annual Demand
1	62	94	113	41	310
2	73	110	130	52	365
3	79	118	140	58	395
4	83	124	146	62	415
5	89	135	161	65	450
6	94	139	162	70	465

ILLUSTRATION OF THE NAÏVE METHOD

Naïve method: The forecast for next period (period t+1) will be equal to this period's actual demand (A_t).

In this illustration we assume that each year (beginning with year 2) we made a forecast, then waited to see what demand unfolded during the year. We then made a forecast for the subsequent year, and so on right through to the forecast for year 7.

Year	Actual Demand (A_t)	Forecast (F_t)	Notes
1	310	--	There was no prior demand data on which to base a forecast for period 1
2	365	310	From this point forward, these forecasts were made on a year-by-year basis.
3	395	365	
4	415	395	
5	450	415	
6	465	450	
7		465	

MEAN (SIMPLE AVERAGE) METHOD

Mean (simple average) method: The forecast for next period (period t+1) will be equal to the average of all past historical demands.

In this illustration we assume that a simple average method is being used. We will also assume that, in the absence of data at startup, we made a guess for the year 1 forecast (300). At the end of year 1 we could start using this forecasting method. In this illustration we assume that each year (beginning with year 2) we made a forecast, then waited to see what demand unfolded during the year. We then made a forecast for the subsequent year, and so on right through to the forecast for year 7.

Year	Actual Demand (A_t)	Forecast (F_t)	Notes
1	310	300	This forecast was a guess at the beginning.
2	365	310.000	From this point forward, these forecasts were made on a year-by-year basis using a simple average approach.
3	395	337.500	
4	415	356.667	
5	450	371.250	
6	465	387.000	
7		400.000	

SIMPLE MOVING AVERAGE METHOD

Simple moving average method: The forecast for next period (period t+1) will be equal to the average of a specified number of the most recent observations, with each observation receiving the same emphasis (weight).

In this illustration we assume that a 2-year simple moving average is being used. We will also assume that, in the absence of data at startup, we made a guess for the year 1 forecast (300). Then, after year 1 elapsed, we made a forecast for year 2 using a naïve method (310). Beyond that point we had sufficient data to let our 2-year simple moving average forecasts unfold throughout the years.

Year	Actual Demand (A_t)	Forecast (F_t)	Notes
1	310	300	This forecast was a guess at the beginning.
2	365	310	This forecast was made using a naïve approach.
3	395	337.500	From this point forward, these forecasts were made on a year-by-year basis using a 2-yr moving average approach.
4	415	380.000	
5	450	405.000	
6	465	432.500	
7		457.500	

ANOTHER SIMPLE MOVING AVERAGE ILLUSTRATION

In this illustration we assume that a 3-year simple moving average is being used. We will also assume that, in the absence of data at startup, we made a guess for the year 1 forecast (300). Then, after year 1 elapsed, we used a naïve method to make a forecast for year 2 (310) and year 3 (365). Beyond that point we had sufficient data to let our 3-year simple moving average forecasts unfold throughout the years.

Year	Actual Demand (A_t)	Forecast (F_t)	Notes
1	310	300	This forecast was a guess at the beginning.
2	365	310	This forecast was made using a naïve approach.
3	395	365	This forecast was made using a naïve approach.
4	415	356.667	From this point forward, these forecasts were made on a year-by-year basis using a 3-yr moving average approach.
5	450	391.667	
6	465	420.000	
7		433.333	

WEIGHTED MOVING AVERAGE METHOD

Weighted moving average method: The forecast for next period (period t+1) will be equal to a weighted average of a specified number of the most recent observations.

In this illustration we assume that a 3-year weighted moving average is being used. We will also assume that, in the absence of data at startup, we made a guess for the year 1 forecast (300). Then, after year 1 elapsed, we used a naïve method to make a forecast for year 2 (310) and year 3 (365). Beyond that point we had sufficient data to let our 3-year weighted moving average forecasts unfold throughout the years. The weights that were to be used are as follows: Most recent year, .5; year prior to that, .3; year prior to that, .2

Year	Actual Demand (A_t)	Forecast (F_t)	Notes
1	310	300	This forecast was a guess at the beginning.
2	365	310	This forecast was made using a naïve approach.
3	395	365	This forecast was made using a naïve approach.
4	415	369.000	From this point forward, these forecasts were made on a year-by-year basis using a 3-yr wtd. moving avg. approach.
5	450	399.000	
6	465	428.500	
7		450.500	

EXPONENTIAL SMOOTHING METHOD

Exponential smoothing method: The new forecast for next period (period t) will be calculated as follows:

New forecast = Last period's forecast + α(Last period's actual demand – Last period's forecast)

(***this box contains all you need to know to apply exponential smoothing)***

$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$ (equation 1)

$F_t = \alpha A_{t-1} + (1-\alpha)F_{t-1}$ (alternate equation 1 – a bit more user friendly)

Where α is a smoothing coefficient whose value is between 0 and 1.

The exponential smoothing method only requires that you dig up two pieces of data to apply it (the most recent actual demand and the most recent forecast).

An attractive feature of this method is that forecasts made with this model will include a portion of every piece of historical demand. Furthermore, there will be different weights placed on these historical demand values, with older data receiving lower weights. At first glance this may not be obvious, however, this property is illustrated on the following page.

DEMONSTRATION: EXPONENTIAL SMOOTHING INCLUDES ALL PAST DATA

Note: the mathematical manipulations in this box are not something you would ever have to do when applying exponential smoothing. All you need to use is equation 1 on the previous page. This demonstration is to convince the skeptics that when using equation 1, all historical data will be included in the forecast, and the older the data, the lower the weight applied to that data.

To make a forecast for next period, we would use the user friendly alternate equation 1:

$F_t = \alpha A_{t-1} + (1-\alpha)F_{t-1}$ **(equation 1)**

When we made the forecast for the current period (F_{t-1}), it was made in the following fashion:

$F_{t-1} = \alpha A_{t-2} + (1-\alpha)F_{t-2}$ (equation 2)

If we substitute equation 2 into equation 1 we get the following:

$F_t = \alpha A_{t-1} + (1-\alpha)[\alpha A_{t-2} + (1-\alpha)F_{t-2}]$

Which can be cleaned up to the following:

$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + (1-\alpha)^2F_{t-2}$ (equation 3)

We could continue to play that game by recognizing that $F_{t-2} = \alpha A_{t-3} + (1-\alpha)F_{t-3}$ (equation 4)

If we substitute equation 4 into equation 3 we get the following:

$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + (1-\alpha)^2[\alpha A_{t-3} + (1-\alpha)F_{t-3}]$

Which can be cleaned up to the following:

$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2A_{t-3} + (1-\alpha)^3F_{t-3}$

If you keep playing that game, you should recognize that

$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2A_{t-3} + \alpha(1-\alpha)^3A_{t-4} + \alpha(1-\alpha)^4A_{t-5} + \alpha(1-\alpha)^5A_{t-6}$ **..........**

As you raise those decimal weights to higher and higher powers, the values get smaller and smaller.

EXPONENTIAL SMOOTHING ILLUSTRATION

In this illustration we assume that, in the absence of data at startup, we made a guess for the year 1 forecast (300). Then, for each subsequent year (beginning with year 2) we made a forecast using the exponential smoothing model. After the forecast was made, we waited to see what demand unfolded during the year. We then made a forecast for the subsequent year, and so on right through to the forecast for year 7.

This set of forecasts was made using an α value of .1

Year	Actual Demand (A)	Forecast (F)	Notes
1	310	300	This was a guess, since there was no prior demand data.
2	365	301	From this point forward, these forecasts were made on a year-by-year basis using exponential smoothing with α=.1
3	395	307.4	
4	415	316.16	
5	450	326.044	
6	465	338.4396	
7		351.09564	

A SECOND EXPONENTIAL SMOOTHING ILLUSTRATION

In this illustration we assume that, in the absence of data at startup, we made a guess for the year 1 forecast (300). Then, for each subsequent year (beginning with year 2) we made a forecast using the exponential smoothing model. After the forecast was made, we waited to see what demand unfolded during the year. We then made a forecast for the subsequent year, and so on right through to the forecast for year 7.

This set of forecasts was made using an α value of .2

Year	Actual Demand (A)	Forecast (F)	Notes
1	310	300	This was a guess, since there was no prior demand data.
2	365	302	From this point forward, these forecasts were made on a year-by-year basis using exponential smoothing with α=.2
3	395	314.6	
4	415	330.68	
5	450	347.544	
6	465	368.0352	
7		387.42816	

A THIRD EXPONENTIAL SMOOTHING ILLUSTRATION

In this illustration we assume that, in the absence of data at startup, we made a guess for the year 1 forecast (300). Then, for each subsequent year (beginning with year 2) we made a forecast using the exponential smoothing model. After the forecast was made, we waited to see what demand unfolded during the year. We then made a forecast for the subsequent year, and so on right through to the forecast for year 7.

This set of forecasts was made using an α value of .4

Year	Actual Demand (A)	Forecast (F)	Notes
1	310	300	This was a guess, since there was no prior demand data.
2	365	304	From this point forward, these forecasts were made on a year-by-year basis using exponential smoothing with α=.4
3	395	328.4	
4	415	355.04	
5	450	379.024	
6	465	407.4144	
7		430.44864	

TREND PROJECTION

Trend projection method: This method is a version of the linear regression technique. It attempts to draw a straight line through the historical data points in a fashion that comes as close to the points as possible. (Technically, the approach attempts to reduce the vertical deviations of the points from the trend line, and does this by minimizing the squared values of the deviations of the points from the line). Ultimately, the statistical formulas compute a slope for the trend line (b) and the point where the line crosses the y-axis (a). This results in the straight line equation

$Y = a + bX$

Where X represents the values on the horizontal axis (time), and Y represents the values on the vertical axis (demand).

For the demonstration data, computations for b and a reveal the following ***(NOTE: I will not require you to make the statistical calculations for b and a; these would be given to you. However, you do need to know what to do with these values when given to you.)***

$b = 30$

$a = 295$

$Y = 295 + 30X$

This equation can be used to forecast for any year into the future. For example:

Year 7: Forecast = 295 + 30(7) = 505

Year 8: Forecast = 295 + 30(8) = 535

Year 9: Forecast = 295 + 30(9) = 565

Year 10: Forecast = 295 + 30(10) = 595

<u>STABILITY VS. RESPONSIVENESS IN FORECASTING</u>

All demand forecasting methods vary in the degree to which they emphasize recent demand changes when making a forecast. Forecasting methods that react very strongly (or quickly) to demand changes are said to be ***responsive.*** Forecasting methods that do not react quickly to demand changes are said to be ***stable.*** One of the critical issues in selecting the appropriate forecasting method hinges on the question of ***stability*** versus ***responsiveness.*** How much stability or how much responsiveness one should employ is a function of how the historical demand has been fluctuating. If demand has been showing a steady pattern of increase (or decrease), then more responsiveness is desirable, for we would like to react quickly to those demand increases (or decreases) when we make our next forecast. On the other hand, if demand has been fluctuating upward and downward, then more stability is desirable, for we do not want to "over react" to those up and down fluctuations in demand.

For some of the simple forecasting methods we have examined, the following can be noted:

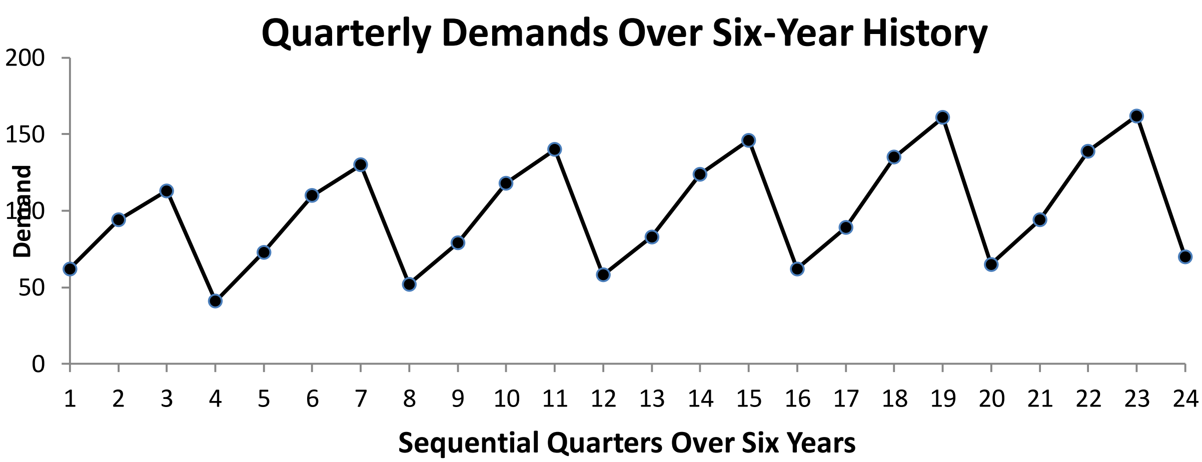
Moving Average Approach: Using more periods in your moving average forecasts will result in more stability in the forecasts. Using fewer periods in your moving average forecasts will result in more responsiveness in the forecasts.

Weighted Moving Average Approach: Using more periods in your weighted moving average forecasts will result in more stability in the forecasts. Using fewer periods in your weighted moving average forecasts will result in more responsiveness in the forecasts. Furthermore, placing lower weights on the more recent demand will result in more stability in the forecasts. Placing higher weights on the more recent demand will result in more responsiveness in the forecasts.

Simple Exponential Smoothing Approach: Using a lower alpha (α) value will result in more stability in the forecasts. Using a higher alpha (α) value will result in more responsiveness in the forecasts.

SEASONALITY ISSUES IN FORECASTING

Up to this point we have seen several ways to make a forecast for an upcoming year. In many instances managers may want more detail that just a yearly forecast. They may like to have a projection for individual time periods within that year (e.g., weeks, months, or quarters). Let's assume that our forecasted demand for an upcoming year is 480, but management would like a forecast for each of the quarters of the year. A simple approach might be to simply divide the total annual forecast of 480 by 4, yielding 120. We could then project that the demand for each quarter of the year will be 120. But of course, such forecasts could be expected to be quite inaccurate, for an examination of our original table of historical data reveals that demand is not uniform across each quarter of the year. There seem to be distinct peaks and valleys (i.e., quarters of higher demand and quarters of lower demand). The graph below of the historical quarterly demand clearly shows those peaks and valleys during the course of each year.



Mechanisms for dealing with seasonality are illustrated over the next several pages.

CALCULATING SEASONAL INDEX VALUES

This is the way you will find seasonal index values calculated in the textbook. Begin by calculating the average demand in each of the four quarters of the year.

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6
Year	Q1	Q2	Q3	Q4	Annual Demand
1	62	94	113	41	310
2	73	110	130	52	365
3	79	118	140	58	395
4	83	124	146	62	415
5	89	135	161	65	450
6	94	139	162	70	465
Avg. Demand Per Qtr.	(62+73+ 79+83+ 89+94) ÷ 6 = 80	(94+110+ 118+124+ 135+139) ÷ 6 = 120	(113+130+ 140+146+ 161+162) ÷ 6 = 142	(41+52+ 58+62+ 65+70) ÷ 6 = 58	

Next, note that the total demand over these six years of history was 2400 (i.e., 310 + 365 + 395 + 415 + 450 + 465), and if this total demand of 2400 had been evenly spread over each of the 24 quarters in this six year period, the average quarterly demand would have been 100 units. Another way to look at this is the average of the quarterly averages is 100 units, i.e. (80 + 120 + 142 + 58)/4 = 100 units.

But, the numbers above indicate that the demand wasn't evenly distributed over each quarter. In Quarter 1 the average demand was considerably below 100 (it averaged 80 in Quarter 1). In Quarters 2 and 3 the average demand was considerably above 100 (with averages of 120 and 142, respectively). Finally, in Quarter 4 the average demand was below 100 (it averaged 58 in Quarter 4). We can calculate a seasonal index for each quarter by dividing the average quarterly demand by the 100 that would have occurred if all the demand had been evenly distributed across the quarters.

This would result in the following alternate seasonal index values:

Year	Q1	Q2	Q3	Q4
Seasonal Index	80/100 = .80	120/100 = 1.20	142/100 = 1.42	58/100 = .58

A quick check of these alternate seasonal index values reveals that they average out to 1.0 (as they should). (.80 + 1.20 + 1.42 + .58)/4 = 1.000

USING SEASONAL INDEX VALUES

The following forecasts were made for the next 4 years using the trend projection line approach (the trend projection formula developed was Y = 295 + 30X, where Y is the forecast and X is the year number).

Year	Forecast
7	505
8	535
9	565
10	595

If these annual forecasts were evenly distributed over each year, the quarterly forecasts would look like the following:

Year	Q1	Q2	Q3	Q4	Annual Forecast	Annual/4
7	126.25	126.25	126.25	126.25	505	126.25
8	133.75	133.75	133.75	133.75	535	133.75
9	141.25	141.25	141.25	141.25	565	141.25
10	148.75	148.75	148.75	148.75	595	148.75

However, seasonality in the past demand suggests that these forecasts should not be evenly distributed over each quarter. We must take these even splits and multiply them by the seasonal index (S.I.) values to get a more reasonable set of quarterly forecasts. The results of these calculations are shown below.

S.I.	.80	1.20	1.42	.58

Year	Q1	Q2	Q3	Q4	Annual Forecast
7	101.000	151.500	179.275	73.225	505
8	107.000	160.500	189.925	77.575	535
9	113.000	169.500	200.575	81.925	565
10	119.000	178.500	211.225	86.275	595

If you check these final splits, you will see that the sum of the quarterly forecasts for a particular year will equal the total annual forecast for that year (sometimes there might be a slight rounding discrepancy).

OTHER METHODS FOR MAKING SEASONAL FORECASTS

Let's go back and reexamine the historical data we have for this problem. I have put a little separation between the columns of each quarter to let you better visualize the fact that we could look at any one of those vertical strips of data and treat it as a time series. For example, the Q1 column displays the progression of quarter 1 demands over the past six years. One could simply peel off that strip of data and use it along with any of the forecasting methods we have examined to forecast the Q1 demand in year 7. We could do the same thing for each of the other three quarterly data strips.

Year	Q1	Q2	Q3	Q4
1	62	94	113	41
2	73	110	130	52
3	79	118	140	58
4	83	124	146	62
5	89	135	161	65
6	94	139	162	70

To illustrate, I have used the linear trend line method on the quarter 1 strip of data, which would result in the following trend line:

$Y = 58.8 + 6.0571X$

For year 7, $X = 7$, so the resulting Q1 forecast for year 7 would be 101.200

We could do the same thing with the Q2, Q3, and Q4 strips of data. For each strip we would compute the trend line equation and use it to project that quarter's year 7 demand. Those results are summarized here:

Q2 trend line: $Y = 89.4 + 8.7429X$; Year 7 Q2 forecast would be 150.600
Q3 trend line: $Y = 107.6 + 9.8286X$; Year 7 Q3 forecast would be 176.400
Q4 trend line: $Y = 39.2 + 5.3714X$; Year 7 Q4 forecast would be 76.800

Total forecast for year 7 = 101.200 + 150.600 + 176.400 + 76.800 = 505.000

These quarterly forecasts are in the same ballpark as those made with the seasonal index values earlier. They differ a bit, but we cannot say one is correct and one is incorrect. They are just slightly different predictions of what is going to happen in the future. They do provide a total annual forecast that is equal to the trend projection forecast made for year 7. (Don't expect this to occur on every occasion, but since it corroborates results obtained with a different method, it does give us confidence in the forecasts we have made.)

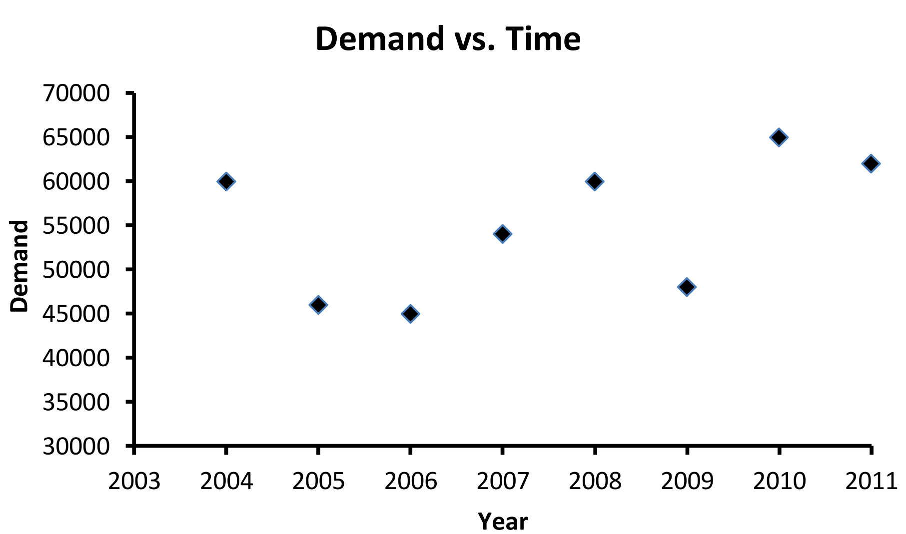
ASSOCIATIVE FORECASTING METHOD

Associative forecasting models (causal models) assume that the variable being forecasted (the dependent variable) is related to other variables (independent variables) in the environment. This approach tries to project demand based upon those associations. In its simplest form, linear regression is used to fit a line to the data. That line is then used to forecast the dependent variable for some selected value of the independent variable.

In this illustration a distributor of drywall in a local community has historical demand data for the past eight years as well as data on the number of permits that have been issued for new home construction. These data are displayed in the following table:

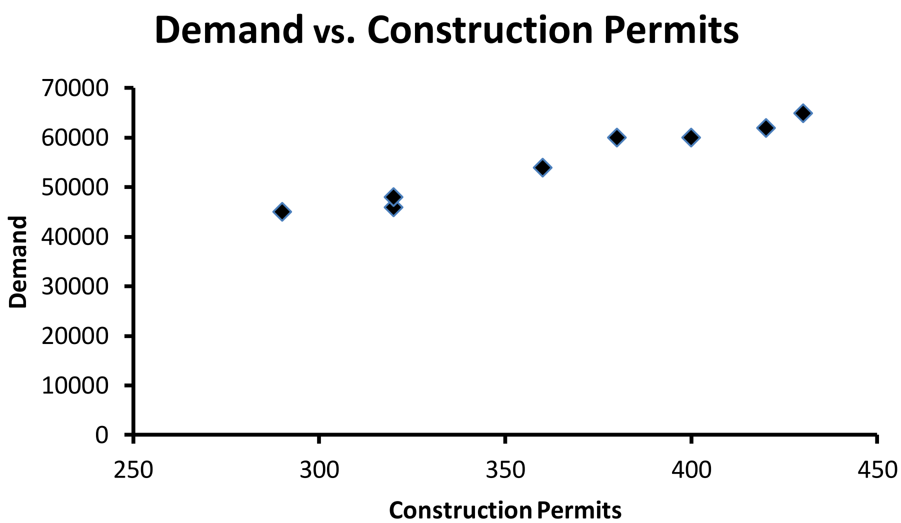
Year	# of new home construction permits	Demand for 4'x8' sheets of drywall
2004	400	60,000
2005	320	46,000
2006	290	45,000
2007	360	54,000
2008	380	60,000
2009	320	48,000
2010	430	65,000
2011	420	62,000

If we attempted to perform a time series analysis on demand, the results would not make much sense, for a quick plot of demand vs. time suggests that there is no apparent pattern relationship here, as seen below.



ASSOCIATIVE FORECASTING METHOD (CONTINUED)

If you plot the relationship between demand and the number of construction permits, a pattern emerges that makes more sense. It seems to indicate that demand for this product is lower when fewer construction permits are issued, and higher when more permits are issued. Therefore, regression will be used to establish a relationship between the dependent variable (demand) and the independent variable (construction permits).



The independent variable (X) is the number of construction permits. The dependent variable (Y) is the demand for drywall.

Application of regression formulas yields the following forecasting model:

Y = 250 + 150X

If the company plans finds from public records that 350 construction permits have been issued for the year 2012, then a reasonable estimate of drywall demand for 2012 would be:

Y = 250 + 150(350) = 250 + 52,500 = 52,750

(which means next year's forecasted demand is 52,750 sheets of drywall)

MEASURING FORECAST ACCURACY

Mean Forecast Error (MFE): Forecast error is a measure of how accurate our forecast was in a given time period. It is calculated as the actual demand minus the forecast, or

$E_t = A_t - F_t$

Forecast error in one time period does not convey much information, so we need to look at the accumulation of errors over time. We can calculate the average value of these forecast errors over time (i.e., a **Mean Forecast Error**, or **MFE**).Unfortunately, the accumulation of the E_t values is not always very revealing, for some of them will be positive errors and some will be negative. These positive and negative errors cancel one another, and looking at them alone (or looking at the MFE over time) might give a false sense of security. To illustrate, consider our original data, and the accompanying pair of hypothetical forecasts made with two different forecasting methods.

Year	Actual Demand A_t	Hypothetical Forecasts Made With Method 1 F_t	Forecast Error With Method 1 $A_t - F_t$	Hypothetical Forecasts Made With Method 2 F_t	Forecast Error With Method 2 $A_t - F_t$
1	310	315	-5	370	-60
2	365	375	-10	455	-90
3	395	390	5	305	90
4	415	405	10	535	-120
5	450	435	15	390	60
6	465	480	-15	345	120
Accumulated Forecast Errors			0		0
Mean Forecast Error, MFE			0/6 = 0		0/6 = 0

Based on the accumulated forecast errors over time, the two methods look equally good. But, most observers would judge that Method 1 is generating better forecasts than Method 2 (i.e., smaller misses).

MEASURING FORECAST ACCURACY

Mean Absolute Deviation (MAD): To eliminate the problem of positive errors canceling negative errors, a simple measure is one that looks at the absolute value of the error (size of the deviation, regardless of sign). When we disregard the sign and only consider the size of the error, we refer to this deviation as the absolute deviation. If we accumulate these absolute deviations over time and find the average value of these absolute deviations, we refer to this measure as the mean absolute deviation (MAD). For our hypothetical two forecasting methods, the absolute deviations can be calculated for each year and an average can be obtained for these yearly absolute deviations, as follows:

		Hypothetical Forecasting Method 1			Hypothetical Forecasting Method 2		
Year	Actual Demand A_t	Forecast F_t	Forecast Error $A_t - F_t$	Absolute Deviation $\lvert A_t - F_t \rvert$	Forecast F_t	Forecast Error $A_t - F_t$	Absolute Deviation $\lvert A_t - F_t \rvert$
1	310	315	-5	5	370	-60	60
2	365	375	-10	10	455	-90	90
3	395	390	5	5	305	90	90
4	415	405	10	10	535	-120	120
5	450	435	15	15	390	60	60
6	465	480	-15	15	345	120	120
	Total Absolute Deviation			60			540
	Mean Absolute Deviation			60/6=10			540/6=90

The smaller misses of Method 1 has been formalized with the calculation of the MAD. Method 1 seems to have provided more accurate forecasts over this six year horizon, as evidenced by its considerably smaller MAD.

MEASURING FORECAST ACCURACY

Mean Squared Error (MSE): Another way to eliminate the problem of positive errors canceling negative errors is to square the forecast error. Regardless of whether the forecast error has a positive or negative sign, the squared error will always have a positive sign. If we accumulate these squared errors over time and find the average value of these squared errors, we refer to this measure as the mean squared error (MSE). For our hypothetical two forecasting methods, the squared errors can be calculated for each year and an average can be obtained for these yearly squared errors, as follows:

Year	Actual Demand A_t	Hypothetical Forecasting Method 1			Hypothetical Forecasting Method 2		
		Forecast F_t	Forecast Error $A_t - F_t$	Squared Error $(A_t - F_t)^2$	Forecast F_t	Forecast Error $A_t - F_t$	Squared Error $(A_t - F_t)^2$
1	310	315	-5	25	370	-60	3600
2	365	375	-10	100	455	-90	8100
3	395	390	5	25	305	90	8100
4	415	405	10	100	535	-120	14400
5	450	435	15	225	390	60	3600
6	465	480	-15	225	345	120	14400
	Total Squared Error			700			52200
	Mean Squared Error			700/6 = 116.67			52200/6 = 8700

Method 1 seems to have provided more accurate forecasts over this six year horizon, as evidenced by its considerably smaller MSE.

The Question often arises as to why one would use the more cumbersome MSE when the MAD calculations are a bit simpler (you don't have to square the deviations). MAD does have the advantage of simpler calculations. However, there is a benefit to the MSE method. Since this method squares the error term, large errors tend to be magnified. Consequently, MSE places a higher penalty on large errors. This can be useful in situations where small forecast errors don't cause much of a problem, but large errors can be devastating.

MEASURING FORECAST ACCURACY

Mean Absolute Percent Error (MAPE): A problem with both the MAD and MSE is that their values depend on the magnitude of the item being forecast. If the forecast item is measured in thousands or millions, the MAD and MSE values can be very large. To avoid this problem, we can use the MAPE. MAPE is computed as the average of the absolute difference between the forecasted and actual values, expressed as a percentage of the actual values. In essence, we look at how large the miss was relative to the size of the actual value. For our hypothetical two forecasting methods, the absolute percentage error can be calculated for each year and an average can be obtained for these yearly values, yielding the MAPE, as follows:

		Hypothetical Forecasting Method 1			Hypothetical Forecasting Method 2		
Year	Actual Demand A_t	Forecast F_t	Forecast Error $A_t - F_t$	Absolute % Error $100\|A_t - F_t\|/A_t$	Forecast F_t	Forecast Error $A_t - F_t$	Absolute % Error $100\|A_t - F_t\|/A_t$
1	310	315	-5	1.16%	370	-60	19.35%
2	365	375	-10	2.74%	455	-90	24.66%
3	395	390	5	1.27%	305	90	22.78%
4	415	405	10	2.41%	535	-120	28.92%
5	450	435	15	3.33%	390	60	13.33%
6	465	480	-15	3.23%	345	120	17.14%
	Total Absolute % Error			14.59%			134.85%
	Mean Absolute % Error			14.59/6= 2.43%			134.85/6= 22.48%

Method 1seems to have provided more accurate forecasts over this six year horizon, as evidenced by the fact that the percentages by which the forecasts miss the actual demand are smaller with Method 1 (i.e., smaller MAPE).

ILLUSTRATION OF THE FOUR FORECAST ACCURACY MEASURES

Here is a further illustration of the four measures of forecast accuracy, this time using hypothetical forecasts that were generated using some different methods than the previous illustrations (called forecasting methods A and B; actually, these forecasts were made up for purposes of illustration). These calculations illustrate why we cannot rely on just one measure of forecast accuracy.

		Hypothetical Forecasting Method A					Hypothetical Forecasting Method B				
Year	Actual Demand A_t	Forecast F_t	Forecast Error $A_t - F_t$	Absolute Deviation $\|A_t - F_t\|$	Squared Deviation $(A_t - F_t)^2$	Abs. % Error $\|A_t-F_t\|/A_t$	Forecast F_t	Forecast Error $A_t - F_t$	Absolute Deviation $\|A_t - F_t\|$	Squared Deviation $(A_t - F_t)^2$	Abs. % Error $\|A_t-F_t\|/A_t$
1	310	330	-20	20	400	6.45%	310	0	0	0	0%
2	365	345	20	20	400	5.48%	365	0	0	0	0%
3	395	415	-20	20	400	5.06%	395	0	0	0	0%
4	415	395	20	20	400	4.82%	415	0	0	0	0%
5	450	430	20	20	400	4.44%	390	60	60	3600	13.33%
6	465	485	-20	20	400	4.30%	525	-60	60	3600	12.90%
		Totals	0	120	2400	30.55%	Totals	0	120	7200	26.23%
			MFE = 0/6 = 0	MAD = 120/6 = 20	MSE = 2400/6 = 400	MAPE= 30.55/6 5.09%		MFE = 0/6 = 0	MAD = 120/6 = 20	MSE = 7200/6 = 1200	MAPE= 26.23/6 4.37%

You can observe that for each of these forecasting methods, the same MFE resulted and the same MAD resulted. With these two measures, we would have no basis for claiming that one of these forecasting methods was more accurate than the other. With several measures of accuracy to consider, we can look at all the data in an attempt to determine the better forecasting method to use. Interpretation of these results will be impacted by the biases of the decision maker and the parameters of the decision situation. For example, one observer could look at the forecasts with method A and note that they were pretty consistent in that they were always missing by a modest amount (in this case, missing by 20 units each year). However, forecasting method B was very good in some years, and extremely bad in some years (missing by 60 units in years 5 and 6). That observation might cause this individual to prefer the accuracy and consistency of forecasting method A. This causal observation is formalized in the calculation of the MSE. Forecasting method A has a considerably lower MSE than forecasting method B. The squaring magnified those big misses that were observed with forecasting method B. However, another individual might view these results and have a preference for method B, for the sizes of the misses relative to the sizes of the actual demand are smaller than for method A, as indicated by the MAPE calculations.